Homework 3: Divide and Conquer

1 The mixture sequence problem

A sequence of n natural numbers $C = \langle c_1, c_2, \ldots, c_n \rangle$ is said to be a ***mixture*** of a sequence of n natural numbers $A = \langle a_1, a_2, \ldots, a_n \rangle$ and another sequence of n natural numbers $B = \langle b_1, b_2, \ldots, b_n \rangle$ if and only if for every $1 \leq i \leq n$ we have $c_i = a_i$ or $c_i = b_i$. In other words, for i from 1 to n, we pick either a_i or b_i and make it c_i. We say $\sum_{1 \leq i \leq n} c_i$ is a ***mixture sum*** of A and B.

The mixture sequence problem is defined as follows:

- Given two sequences of n natural numbers of $A = \langle a_1, a_2, \ldots, a_n \rangle$ and $B = \langle b_1, b_2, \ldots, b_n \rangle$,
- and given a natural number m,
- determine whether there exists a mixture C of A and B such that the the mixture sum of C equals m, and
- if such a mixture sequence does exist print out the mixture sequence; otherwise print out a message saying there is no such mixture sequence.

2 A Divide-and-Conquer Approach

The following is a possible divide-and-conquer approach to solve the problem:

- Assume n is a power of 2. (Note that you can always extend both sequences to the length of the closest power of 2 by filling in zeros in the sequences.) Divide the sequence $A = \langle a_1, a_2, \ldots, a_n \rangle$ into two sequences of size $n/2$: $A_1 = \langle a_1, a_2, \ldots, a_{n/2} \rangle$ and $A_2 = \langle a_{n/2+1}, a_{n/2+2}, \ldots, a_n \rangle$ and $B = \langle b_1, b_2, \ldots, b_n \rangle$ into two sequences of size $n/2$: $B_1 = \langle b_1, b_2, \ldots, b_{n/2} \rangle$ and $B_2 = \langle b_{n/2+1}, b_{n/2+2}, \ldots, b_n \rangle$
- For each integer m where $0 \leq m' \leq m$, recursively solve (i) a smaller instance of the mixture sequence problem with the sequences A_1 and B_1 and with the target mixture sum m' and (ii) a smaller instance of the mixture sequence problem with the sequences A_2 and B_2 and with the target mixture sum $m - m'$. If for any particular m' where $0 \leq m' \leq m$, there are solutions C_1 and C_2 to the two problem instances as described above, then concatenate C_1 and C_2 into a sequence C and C is a solution to the original problem instance. If there is no such m', then there is no solution to the original problem instance.

3 The Programming Assignment

Implement the divide-and-conquer approach above or some other divide-and-conquer approach of your own into a program. Your program should interact with the user like what you did in Homework 1. Check and report how much time your programs for Homework 1 and Homework 1 and your new program for this homework will take respectively over the problem instances posted on our class web site. Report the solutions found by your program.