Homework #1B: Exhaustive search for solving the mixture-sum problem

(Brute-force search through a search space of \(2^n\) possibilities)

A sequence of \(n\) natural numbers \(C = \langle c_1, c_2, \ldots, c_n \rangle\) is said to be a mixture of a sequence of \(n\) natural numbers \(A = \langle a_1, a_2, \ldots, a_n \rangle\) and another sequence of \(n\) natural numbers \(B = \langle b_1, b_2, \ldots, b_n \rangle\) if and only if for every \(1 \leq i \leq n\) we have \(c_i = a_i\) or \(c_i = b_i\). In other words, for \(i\) from 1 to \(n\), we pick either \(a_i\) or \(b_i\) and make it \(c_i\). We say \(\sum_{1 \leq i \leq n} c_i\) is a mixture sum of \(A\) and \(B\).

Write a program that can do the following things:

- read two sequences of \(n\) natural numbers of \(A = \langle a_1, a_2, \ldots, a_n \rangle\) and \(B = \langle b_1, b_2, \ldots, b_n \rangle\) where \(n\) is given by the user during the runtime,
- ask for a natural number \(m\) given by the user during the runtime,
- determine whether there exists a mixture \(C\) of \(A\) and \(B\) such that the the mixture sum of \(C\) equals \(m\), and
- if such a mixture sequence does exist print out the mixture sequence; otherwise print out a message saying there is no such mixture sequence.

For this programming assignment, just conduct an exhaustive search to examine all \(2^n\) possible mixture sequences of \(A\) and \(B\) to see whether there is one with a mixture sum equals \(m\). You can use dynamically allocate a vector of \(n\) integers to simulate an \(n\)-bit binary counter counting from \(\langle 0, 0, \ldots, 0 \rangle\) to \(\langle 1, 1, \ldots, 1 \rangle\) increased by one in each step. In each step, the value of the \(n\)-bit binary counter represents a unique mixture sequence \(C = \langle c_1, c_2, \ldots, c_n \rangle\) where \(c_i = a_i\) if the \(i\)th bit of the binary counter is 0 and \(c_i = b_i\) if the \(i\)th bit of the binary counter is 1.

Example 1:

A: \(<69 38 46 43 37 34 28 75>\)

B: \(<64 77 55 24 69 12 22 69>\)

Is there a mixture sequence \(C\) from A and B with a sum of 400?

Solution: Yes. \(C: <64 38 55 43 69 34 22 75>\)

Test case 1A: Is there a mixture sequence \(C\) from A and B with a sum of 310?

Test case 1B: Is there a mixture sequence \(C\) from A and B with a sum of 312?

Test case 1C: Is there a mixture sequence \(C\) from A and B with a sum of 450?
Test case 1D: Is there a mixture sequence C from A and B with a sum of 453?

Example 2:
A: <61 27 43 54 37 45 28 64 60 38 40 43 37 34 28 75 62 33 43 60 >
B: <75 74 44 24 58 12 33 69 64 70 55 24 69 12 22 69 69 74 38 24 >

Test case 2A:  Is there a mixture sequence C from A and B with a sum of 1000?
Test case 2B:  Is there a mixture sequence C from A and B with a sum of 750?
Test case 2C:  Is there a mixture sequence C from A and B with a sum of 1755?
Test case 2D:  Is there a mixture sequence C from A and B with a sum of 1036?
Test case 2E:  Is there a mixture sequence C from A and B with a sum of 1150?

Things to report in the self-evaluation report:

- For each test case associated with the two examples above, please report the answer your program finds.
- How much time on average does it take roughly to run a test case for example 1? How much time on average does it take roughly to run a test case for example 2?