Homework #1B: Exhaustive search for solving the mixture-sum problem

(Brute-force search through a search space of 2^n possibilities)

A sequence of n natural numbers $C = \langle c_1, c_2, \ldots, c_n \rangle$ is said to be a ***mixture*** of a sequence of n natural numbers $A = \langle a_1, a_2, \ldots, a_n \rangle$ and another sequence of n natural numbers $B = \langle b_1, b_2, \ldots, b_n \rangle$ if and only if for every $1 \leq i \leq n$ we have $c_i = a_i$ or $c_i = b_i$. In other words, for i from 1 to n, we pick either a_i or b_i and make it c_i. We say $\sum_{1 \leq i \leq n} c_i$ is a ***mixture sum*** of A and B.

Write a program that can do the following things:

- read two sequences of n natural numbers of $A = \langle a_1, a_2, \ldots, a_n \rangle$ and $B = \langle b_1, b_2, \ldots, b_n \rangle$ where n is given by the user during the runtime,
- ask for a natural number m given by the user during the runtime,
- determine whether there exists a mixture C of A and B such that the the mixture sum of C equals m, and
- if such a mixture sequence does exist print out the mixture sequence; otherwise print out a message saying there is no such mixture sequence.

For this programming assignment, just conduct an exhaustive search to examine all 2^n possible mixture sequences of A and B to see whether there is one with a mixture sum equals m. You can use dynamically allocate a vector of n integers to simulate an n-bit binary counter counting from $\langle 0, 0, \ldots, 0 \rangle$ to $\langle 1, 1, \ldots, 1 \rangle$ increased by one in each step. In each step, the value of the n-bit binary counter represents a unique mixture sequence $C = \langle c_1, c_2, \ldots, c_n \rangle$ where $c_i = a_i$ if the ith bit of the binary counter is 0 and $c_i = b_i$ if the ith bit of the binary counter is 1.

Example 1:

A: <69 38 46 43 37 34 28 75>

B: <64 77 55 24 69 12 22 69>

Is there a mixture sequence C from A and B with a sum of 400?

Solution: Yes. C: <64 38 55 43 69 34 22 75>

Test case 1A: Is there a mixture sequence C from A and B with a sum of 310?

Test case 1B: Is there a mixture sequence C from A and B with a sum of 312?

Test case 1C: Is there a mixture sequence C from A and B with a sum of 450?

Test case 1D: Is there a mixture sequence C from A and B with a sum of 453?

Example 2:

A: <61 27 43 54 37 45 28 64 60 38 40 43 37 34 28 75 62 33 43 60 >

B: <75 74 44 24 58 12 33 69 64 70 55 24 69 12 22 69 69 74 38 24 >

Test case 2A: Is there a mixture sequence C from A and B with a sum of 1000?

Test case 2B: Is there a mixture sequence C from A and B with a sum of 750?

Test case 2C: Is there a mixture sequence C from A and B with a sum of 1755?

Test case 2D: Is there a mixture sequence C from A and B with a sum of 1036?

Test case 2E: Is there a mixture sequence C from A and B with a sum of 1150?

Things to report in the self-evaluation report:

- For each test case associated with the two examples above, please report the answer your program finds.
- How much time on average does it take roughly to run a test case for example 1? How much time on average does it take roughly to run a test case for example 2?