Reasoning about Probabilities of Transitions and Observations
Part I: Basics of Using HMMs for Spelling Recognition

Hidden Markov Models (HMMs): In Rabiner's tutorial paper on HMMs, Rabiner desribes a HMM of n hidden states $\{S_1, S_2, \ldots, S_n\}$ and m possible observations $\{v_1, v_2, \ldots, v_m\}$ in terms of (π , A , B) where (i) π is an n-by-1 vector encoding the initial-state probabilities, (ii) A is an n-by-n matrix encoding the state transition probabilities, and (iii) B is an n-by-m matrix encoding the observation probabilities. More sepcficially,

- $\pi = < \pi_1, \pi_2, \ldots, \pi_n >$ and each π_i in π represents the probability of having S_i as the initial state.
- A is an n-by-n matrix and each element a_{ij} in A represents the transition probability from S_i to S_j (i.e. the probability of ending in S_j as the next state when the current state is S_i).
- B is an n-by-m matrix and each element b_{ij} in B represents the probability of observing v_j in state S_i (i.e. the probability of observing in v_j when the current state is S_i).

Spelling recognition and the spelling model: The diagrapm below depcits the transition probabilities of a spelling model regarding how a particular person P may type a 3-character word *abc* as depiced in our class handout on HMMs for spelling recognition. The information provided by the diagram correspond to the information encoded in vector π and matrix A in Rabiner's tutorial paper on HMMs where Rabiner describes a HMM in terms of (π , A , B).

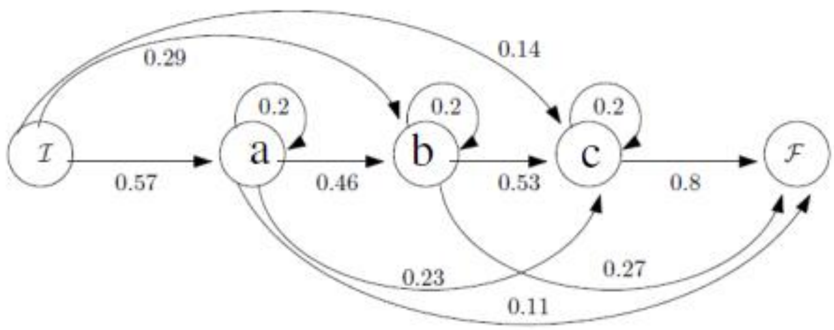

Figure 1: The spelling model regarding the word 'abc' with the parameters $deg_{sp} = 2$, $p_{repeat} = 0.2$, and $p_{moveOn} = 0.8$

Question #1: There are 5 hidden states { S_1=*I*, S_2=*a*, S_3=*b*, S_4=c, S_5=*F*}. In other words, *n*=5 is the number of hidden states in this case. **What is the contents of the corresponding vector π (as a 1 x 5 row vector)? What is the contents of the corresponding matrix A (as a 5 x 5 row matrix)?**

Spelling recognition and the keyboard model:
In the following, let's use a simplied 1-dimensioal keyboard of only 4 keys a, b, c, d as depiced in the simplified example in our class handout on HMMs for spelling recognition. The information provided by tsuch keyboard mdeol corresponds to the information encoded in matrix B in Rabiner's tutorial paper on HMMs where Rabiner describes a HMM in terms of (π , A , B).

A simplified example: Consider the situation that p_{miss} = 0.1, p_{hit} = 0.9, and deg_{kb} = 2. If the one-dimensional keyboard only has 4 keys a, b, c, d (instead of the full 26 keys), the probabilities of typographic mistakes when trying to type a are

- $Pr(Char = b|State = a) = 0.04$,
- $Pr(Char = c|State = a) = 0.02$, and
- $Pr(Char = d|State = a) = 0.04$.

Question #2: We may observe any of the four possible characters each time the person tries to type one character given the 4-keys 1-dimensional keyboard. In the end of the However, for convenience, let's add (i) one sepcial obervarion *ReadyToType* dedicated solely to the special state S_1=*I* and (ii) one sepcial obervarion *EndOfWord* dedicated solely to the special state S_5=*F*. This is because we know the perosn is ready to start the typing process for the word when we are in the special starting state *I*. Similarly we know it is the end of the typing process for the word when we are in the special final state *F*. In other words, (i) the special state S_1=*I* is asscoaited with the special observation *ReadyToType* with probability equal to 1 and (ii) the special state S_1=*F* is asscoaited with the special observation *EndOfWord* with probability equal to 1. So we have *6* possible observations { v_1=*ReadyToType*, v_2=*a*, v_3=*b*, v_4=c, v_5=d, v_6=*EndOfWord*}. In other words, *m*=6 is the number of different obervations we may encounter. **What is the contents of the corresponding matrix B (as a 5 x 6 matrix)?**

Reasoning about Probabilities of Transitions and Observations:

There are many possibilities regarding the process of state transitions and the resulting characters observed when the person tries to type the word *abc* according to the spelling model and the keyboard model described above. For example, the person's mind may go through the following state sequence *Q: I➔a➔b➔b➔F* and produces observation sequence *O*: *ReadyToType➔b➔b➔d➔EndOfWord*, which corresponds to the resulting character string *bbd* observed. How likely could this happen? Equations 12 to 15 on page 262 in Rabiner's tutorial paper show how we can cacluate the joint probability of going through a given state sequence *Q* and seeing a sequence of observations *O*.

Question #3:

What is the probability of

going through a given state sequence *Q: I➔a➔b➔b➔F* and

seeing a sequence of observations *O: ReadyToType➔b➔b➔d➔EndOfWord* ?

Reasoning about Probabilities of Observations:

As discussed above, there are many possibilities regarding the process of state transitions and the resulting characters observed when the person tries to type the word *abc* according to the spelling model and the keyboard model described above. How likely could the person end in the character string *bbd* (i.e. observation sequence *O*: *ReadyToType➔b➔b➔d➔EndOfWord*)?

Note that any feasible state sequence *Q: I➔?➔?➔?➔F* of length 5 may lead to the observation sequence. Equations 16 and 17 on page 262 in Rabiner's tutorial paper tell us that we may (i) enumerate every possible state sequence *Q* and for each *Q* cacluate the joint probability of going through the state sequence *Q and* seeing a sequence of observations *O* and (ii) sum up all joint probabilities in (i) as the result.

Question #4:

Enumerate and show all the possible state sequence *Q: I➔?➔?➔?➔F*.

Question #5:

What is the probability of seeing the character string *bbd* (i.e. observation sequence *O*: *ReadyToType➔b➔b➔d➔EndOfWord*) when the person tries to type the word *abc*?

Note: A more efficient algorithm for finding the answer to Question 5 is the forward algorithm described in pages 262~263 in Rabiner's tutorial paper.