## Homework #2C: Reasoning about HMMs

## Part A

For Part A, answer the questions below in Part A based on the same HMM λ =(π , A , B) that encodes (i) the exact same spelling model regarding a specfic person P trying to type the word *abc* in Homework 2A and shown in the Figure 1 below and (ii) the exact same 4-key one-dimensional keyboard depicted in Homework 2A and also shown in the example below.

**The spelling model for the word *abc***:

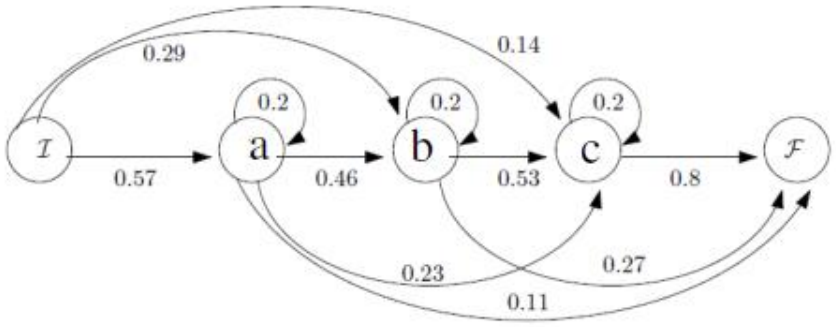


Figure 1: The spelling model regarding the word "abc" with the parameters $deg_{sp} = 2$, $p_{repeat} = 0.2$, and $p_{moveOn} = 0.8$

**One dimensional 4-key keyboard model**:

***A simplified example***: Consider the situation that $p_{miss} = 0.1$, $p_{hit} = 0.9$, and $deg_{kb} = 2$. If the one-dimensional keyboard only has 4 keys $a, b, c, d$ (instead of the full 26 keys), the probabilities of typographic mistakes when trying to type $a$ are

- $Pr(Char = b|State = a) = 0.04$,
- $Pr(Char = c|State = a) = 0.02$, and
- $Pr(Char = d|State = a) = 0.04$.

**Question #A1:** 1 point

What is the probability Pr( Q | λ) of

going through a given state sequence *Q: I➔b➔b➔F* ?

**Question #A2:** 1 point

What is the probability Pr( O | Q, λ) of
seeing a sequence of observations *O:* *ReadyToType➔d➔b➔EndOfWord*
**given that** the underlying state sequence *Q* is *I➔b➔b➔F* ?

**Question #A3:** 1 point
What is the probability Pr( O, Q | λ) of
going through a given state sequence *Q: I➔b➔b➔F*
**and**
seeing a sequence of observations *O: ReadyToType➔d➔b➔EndOfWord* ?

**Question #A4:** 1 point
Enumerate and show all the possible state sequence *Q: I➔?➔?➔F*.

**Question #A5:** 3 points
Use the approach is equation 16 in Rabiner's paper to determine the probability
Pr( O | λ) of seeing the character string *db* (i.e. observation sequence
*O*: *ReadyToType➔d➔b➔EndOfWord* ) when the person tries to type the word *abc*.

**Note:** Instead of putting down a final answer only, record the intermediate results of your work so that you still can get some partial credit if you make mistakes in the process of calculating the answer.

**Question #A6:** 3 points
Use the forward algorithm to determine the probability
Pr( O | λ) of seeing the character string *db* (i.e. observation sequence
*O*: *ReadyToType➔d➔b➔EndOfWord* ) when the person tries to type the word *abc*.

**Note:** You need to show the entire table of your work instead of just giving an answer.

## Part B

For Part B, answer the questions below based on the new HMM λ' =(π' , A' , B') that encodes (i) the spelling model regarding the same person *P* trying to type the word ***ab*** and (ii) the exact same 4-key one-dimensional keyboard depicted in Part A.

**The spelling model for the word *ab***:

For Part B, please consider a **2-character word *ab*** instead of the 3 character word ***abc*** used in Part A, **assuming all the parameter values remain the same, i.e. $deg_{sp}$=2, $p_{repeat}$=0.2, $p_{moveOn}$=0.8 since it is about the typing behavior of the same person**.

Question #B1: 4 points

Draw the transition diagrapm depicting the spelling model regarding how the same person *P* may type a 2 character word *ab* (similar to the way it is depiced in Figure 1 in Part A), **assuming all the parameter values remain the same, i.e. $deg_{sp}$=2, $p_{repeat}$=0.2, $p_{moveOn}$=0.8**.

Note: You may use **paint** or other software to draw the diagram and upload it as a separate image file of bmp or jpg or png format.

Question #B2: 3 points

The information provided by the transition diagram corresponds to the information encoded in vector π and matrix A in Rabiner's tutorial paper on HMMs where Rabiner describes a HMM in terms of λ' = (π' , A' , B'). There are *4* hidden states { $S_1$=*I*, $S_2$=*a*, $S_3$=*b*, $S_4$=*F*}. In other words, *n*=4 is the number of hidden states in this case. **What is the contents of the corresponding vector π' (as a 1 x 4 row vector)? What is the contents of the corresponding matrix A' (as a 4 x 4 row matrix)?**

**One dimensional 4-key keyboard model**:

The person still uses the same one dimensional keyboard depicted in Part A.

> ***A simplified example***: Consider the situation that $p_{miss} = 0.1$, $p_{hit} = 0.9$, and $deg_{kb} = 2$. If the one-dimensional keyboard only has 4 keys $a, b, c, d$ (instead of the full 26 keys), the probabilities of typographic mistakes when trying to type $a$ are
>
> - $Pr(Char = b|State = a) = 0.04$,
> - $Pr(Char = c|State = a) = 0.02$, and
> - $Pr(Char = d|State = a) = 0.04$.

**Question #B3:** 3 points

The information provided by the keyboard mdeol corresponds to the information encoded in matrix B' in Rabiner's tutorial paper on HMMs where Rabiner describes a

HMM in terms of λ' =(π' , A' , B'). **What is the contents of the corresponding matrix B' (as a 4 x 6 matrix)?**

**Note**: We may observe any of the four possible characters each time the person tries to type one character given the 4-keys 1-dimensional keyboard. For convenience as in Homework 2A, let's add (i) one sepcial obervarion *ReadyToType* dedicated solely to the special state $S_1=I$ and (ii) one sepcial obervarion *EndOfWord* dedicated solely to the special state $S_5=F$. In other words, (i) the special state $S_1=I$ is associated with the special observation *ReadyToType* with probability equal to 1 and (ii) the special state $S_1=F$ is assciated with the special observation *EndOfWord* with probability equal to 1. So we have *6* possible observations { $v_1$=*ReadyToType*, $v_2$=*a*, $v_3$=*b*, $v_4$=c, $v_5$=d, $v_6$=*EndOfWord*}. In other words, *m*=6 is the number of different obervation symbols we may encounter. That is why B' is a 4 x 6 matrix.

## Part C

For Part C, answer the questions below. They are the same as those in Part A, but are now **based on the new HMM λ' =(π' , A' , B')** depicted in Part B above that encodes (i) the spelling model regarding a person trying to type the word ***ab*** and (ii) the 4-key one-dimensional keyboard depicted in Part B.

**Question #C1:** 1 point
What is the probability Pr( Q | λ') of
going through a given state sequence *Q: I➔b➔b➔F* ?

**Question #C2:** 1 point
What is the probability Pr( O | Q, λ') of
seeing a sequence of observations *O: ReadyToType➔d➔b➔EndOfWord*
**given that** the underlying state sequence *Q* is *I➔b➔b➔F* ?

**Question #C3:** 1 point
What is the probability Pr( O, Q | λ') of
going through a given state sequence *Q: I➔b➔b➔F*
**and**
seeing a sequence of observations *O: ReadyToType➔d➔b➔EndOfWord* ?

**Question #C4:** 1 point
Enumerate and show all the possible state sequence *Q: I➔?➔?➔F*.

**Question #C5:** 3 points

Use the approach is equation 16 in Rabiner's paper to determine the probability Pr( O | λ') of seeing the character string *db* (i.e. observation sequence *O*: *ReadyToType*➔*d*➔*b*➔*EndOfWord* ) when the person tries to type the word ***ab***.

**Note:** Instead of putting down a final answer only, record the intermediate results of your work so that you still can get some partial credit if you make mistakes in the process of calculating the answer.

**Question #C6:** 3 points

Use the forward algorithm to determine the probability Pr( O | λ') of seeing the character string *db* (i.e. observation sequence *O*: *ReadyToType*➔*d*➔*b*➔*EndOfWord* ) when the person tries to type the word *abc*.

**Note:** You need to show the entire table of your work instead of just giving an answer.

## Part D

**Question #D1:** 2 points

When we observe the string ***db*** on the screen as the result of typing either the word *abc* or the word *ab* (by the same person P modeled in Part A through Part C), are you going to reccognize it as the word *abc* or the word *ab* for spelling recognition purpose? Explain why based on your answers to A5 and A6, C5 and C6.