Homework# 2

1. [Sum rule and product rule, and tree diagrams]: A class has 9 males and 10 females. Determine the number of ways the class can elect **(i)** a class representative; **(ii)** two class representatives of one male and one female; and **(iii)** a class president and a vice president.

2. [Permutation, combination, and tree diagrams]: Determine the number of ways to **(i)** divide 12 students into 3 teams of 5 students, 4 students, and 3 students respectively, **(ii)** divide 12 students into 3 teams of 6 students, 3 students, and 3 students respectively, and **(iii)** divide 12 students into 3 teams of 4 students, 4 students, and 4 students respectively.

3. [Inclusion-exclusion principle]: Consider the specific sets B, B1, B2, B3, and B4 defined in Problem #1 and Problem #2 in Homework #1 and the fact that B is the union of B, B1, B2, B3, and B4 as pointed out in Problem #6 in Homework #1. **(i)** Based on the results you got from Problems #2~#5 in Homework #1 and the facts about the intersections among B1, B2, B3, and B4 indicated in Problem #6 in Homework #1, please apply the equation in Problem #8 in Homework #1 (i.e. the inclusion-exclusion principle for counting the number of elements in the union of 4 sets) to determine the number of the elements in the union of B1, B2, B3, and B4. **(ii)** Is the answer you got above the same as the actual number of elements in B?

4. [Inclusion-exclusion principle]: Consider all the integers from 1 up to and include 1000. Find the number of them that are divisible by **(i)** at least one of 3,5,7; **(ii)** 3 and 5 but not by 7; **(iii)** by 5 but not by 3 nor 7; **(iv)** by none of 3, 5, 7.

5. [Pigeonhole principle]: Prove that for any set X of at least 5 distinct integers there must exist two numbers m and n in X such that m>n and either $m+n$ or $m-n$ is divisible by 7.

6. [Binomial theorem]: Use the binomial theorem to answer the following questions:

(i) What is the coefficient of x^3 in the expansion of $(x+1)^{100}$?

(ii) What is the coefficient of x^{97} in the expansion of $(x+1)^{100}$?

(iii) What is the coefficient of x^3 in the expansion of $(x+2)^{100}$?

7. [Binomial coefficients and binomial theorem]:

(i) For any natural number n, $1 \le n$ and any given natural number i, $1 \le i \le n$, shows that $i*C(n,i) = n*C(n-1,i-1)$.

(ii) Based on the result in (i), show that for any given natural number n we have $1*C(n,1)+2*C(n,2)+ 3*C(n,3)+\ldots.+ n* C(n,n) = n*2^{n-1}$.