Discrete Structures: Homework #1

1. [2 points] Given a year X as a natural number, John only considers X as a leap year **when (and only when)** X fits into ***any one*** of the following descriptions:

(i) ***X is divisible by 4*** and ***X is divisible by 400***

(ii) *X is divisible by 4* and *X is not divisible by 100*.

Let ***D4***, ***D100*** and ***D400*** denote the atomic propositions regarding whether X is divisible by 4, by 100, and by 400 respectively. Please translate John's way of determining whether X is a leap year into a compound logic proposition involving the atomic propositions above, parentheses if needed, and the logic operators $\wedge$, $\vee$, and $\neg$.

2. [2 points] Consider the compound proposition you have for problem#1 above. Is the compound proposition true when X is 1776? How about when X is 1800, 1945, or 2000 respectively?

3. [2 points] Given a year X as a natural number, Mary considers X as a leap year **when (and only when)** X fits into ***none*** of the following descriptions:

(i) *X is not divisible by 4*,

(ii) *X is divisible by 100 and X is not divisible by 400*.

Let ***D4***, ***D100*** and ***D400*** denote the atomic propositions regarding whether X is divisible by 4, by 100, and by 400 respectively. Please translate Mary's way of determining whether X is a leap year into a compound logic proposition involving the atomic propositions above, parentheses if needed, and the logic operators $\wedge$, $\vee$, and $\neg$.

4. [2 points] Consider the compound proposition you have for problem#3 above. Is the compound proposition true when X is 1776? How about when X is 1800, 1945, or 2000 respectively?

5. [2 points] Let's refer to the compound proposition you got in Problem #1 as John's proposition ***J*** and refer to the compound proposition you got in Problem #3 as Mary's proposition ***M***. Create a truth table to check whether ***J*** and ***M*** are logically equivalent in all situations. Are they equivalent based on your finding? Why or why not?