Discrete Structures: Homework #2

1. [3 points] Consider problem #5 in Homework #1. If you determine ***J*** and ***M*** are logically equivalent, now instead of using the truth table please apply **the algebraic laws in Table 6 of Section 1.3 to show the logic equivalence.** You should state the laws and rules you apply in your steps. (Note: If you think they are not equivalent, you can skip this problem and still get the full credit of it if indeed they are not equivalent.)

2. [9 points] Consider the following three compound propositions:
   - i. $(p \rightarrow q) \wedge (\neg p \rightarrow \neg q) \wedge (q \rightarrow \neg p) \wedge (\neg q \rightarrow \neg p)$
   - ii. $(p \rightarrow q) \wedge (\neg p \rightarrow \neg q) \wedge (q \rightarrow \neg p) \wedge (\neg q \rightarrow \neg p) \wedge (p \vee q)$
   - iii. $(p \rightarrow q) \wedge (\neg p \rightarrow \neg q) \wedge (q \rightarrow \neg p) \wedge (\neg q \rightarrow \neg p) \vee (p \vee q)$

   For each of them, use a truth table to determine whether it is a tautology, a contradiction, or a contingency.

3. [9 points] Like in Problem #1 above, please apply **the algebraic laws in Table 6 of Section 1.3** to prove the following. (Note that you need to state the laws you apply in your steps.)
   - i. $(\neg p \vee q) \wedge (p \vee \neg q) \wedge (\neg q \vee \neg p) \wedge (q \vee \neg p)$ is logically equivalent to $(\neg p \wedge \neg q)$.
   - ii. $(\neg p \vee q) \wedge (p \vee \neg q) \wedge (\neg q \vee \neg p) \wedge (q \vee \neg p) \wedge (p \vee q)$ is logically equivalent to F.
   - iii. $(\neg p \vee q) \wedge (p \vee \neg q) \wedge (\neg q \vee \neg p) \wedge (q \vee \neg p) \vee (p \vee q)$ is logically equivalent to T.

4. [3 points] An island has two kinds of inhabitants, *knights*, who always tell the truth, and *knaves*, who always lie. On the island, you encounter two people, A and B. Person A says "B is a knave." Person B says "At least one of us is a knight." Solve the puzzle according to the line of thoughts in the following: **(i)** Let's use p to denote the proposition that A is a knight and let's use q to denote the proposition that B is a knight. **(ii)** According to what A says we know that both $(p \rightarrow \neg q)$ and $(\neg p \rightarrow q)$ need to be true in the end. **(iii)** According to what B says we know that both $(q \rightarrow (p \vee q))$ and $(\neg q \rightarrow \neg (p \vee q))$ need to be true in the end. **(iv)** Therefore finding a solution to the puzzle is simply finding a way of assigning truth values to p and q such that the compound proposition $(p \rightarrow \neg q) \wedge (\neg p \rightarrow q) \wedge (q \rightarrow (p \vee q)) \wedge (\neg q \rightarrow \neg (p \vee q))$ is true.

   In other words, we need to determine whether the compound proposition

   $(p \rightarrow \neg q) \wedge (\neg p \rightarrow q) \wedge (q \rightarrow (p \vee q)) \wedge (\neg q \rightarrow \neg (p \vee q))$ is satisfiable or not.

   Note that you can apply **(i) the algebraic laws in Table 6 of Section 1.3, and/or (ii) the first rule in Table 7 of Section 1.3, and/or (iii) the first rule in Table 8 of Section 1.3** to simplify the compound proposition to an equivalent compound proposition first if you think that is helpful. Then use a truth table to help you determine whether the compound proposition is satisfiable. If it is, also find out a solution (i.e. the truth values assigned to **p** and **q** respectively) that makes the compound proposition true.