

Discrete Structures: Homework #3

1. [6 points] Consider the predicate $P(x, y)$ where (i) the domain for the variables x and y is $\{1, 2, 3\}$ and (ii) only $P(1, 3)$, $P(2, 1)$, $P(2, 2)$, $P(2, 3)$, $P(3, 1)$, $P(3, 2)$ are true while $P(x, y)$ is false otherwise. Determine whether each of the following statements is true.
- $\forall x \exists y P(x, y)$.
 - $\exists x \forall y P(x, y)$.
 - $\neg \exists x \exists y (P(x, y) \wedge \neg P(y, x))$.
 - $\forall y \exists x (P(x, y) \rightarrow P(y, x))$.
 - $\forall x \forall y (x \neq y \rightarrow (P(x, y) \vee P(y, x)))$.
 - $\forall y \exists x (x \leq y \wedge P(x, y))$.

2. [6 points] The domain of variable x below is the set of all students while the domain of variable y is the set of courses. Consider the following predicates about x and/or y :

$U(y)$: y is an upper-level course $M(y)$: y is a math course
 $A(x)$: x is a part-time student $B(x)$: x is a full-time student
 $F(x)$: x is a freshman $T(x, y)$: student x is taking course y .

For each of the following statements, translate them into statements in predicate logic using the variables and the predicates above plus any needed quantifiers.

- Every student is taking at least one course.
 - There is a part-time student who is not taking any math course.
 - Every part-time freshman is taking some upper-level course.
3. [22 points] Consider the following three English sentences:
- Some freshmen are math majors.
 - Every math major is a freshman.
 - No math major is a freshman.

Also consider the following statements in predicate logic:

- $\forall x (M(x) \rightarrow \neg F(x))$.
- $\neg \exists x (M(x) \wedge \neg F(x))$.
- $\forall x (F(x) \rightarrow \neg M(x))$.
- $\forall x (M(x) \rightarrow F(x))$.
- $\exists x (F(x) \wedge M(x))$.
- $\neg \forall x (\neg F(x) \vee \neg M(x))$.
- $\forall x (\neg(M(x) \wedge \neg F(x)))$.
- $\forall x (\neg M(x) \vee \neg F(x))$.
- $\neg \exists x (M(x) \wedge \neg F(x))$.
- $\neg \exists x (M(x) \wedge F(x))$.
- $\neg \forall x (F(x) \rightarrow \neg M(x))$.

The domain of variable x above is the set of all students while two predicates involved are

$F(x)$: x is a freshman. $M(x)$: x is a math major.

For each of the 11 statements in predicate logic, identify the statement with one of the three English sentences as an equivalent translation in English.