

Homework# 7

- [Product rule, sum rule, and tree diagrams]: A class has 9 males and 10 females. Determine the number of ways the class can elect **(i)** a class representative; **(ii)** two class representatives of one male and one female; and **(iii)** a class president and a vice president.
- [Product rule, sum rule, division rule, and tree diagrams]: Determine the number of ways to **(i)** divide 12 students into 3 teams of 5 students, 4 students, and 3 students respectively, **(ii)** divide 12 students into 3 teams of 6 students, 3 students, and 3 students respectively, and **(iii)** divide 12 students into 3 teams of 4 students, 4 students, and 4 students respectively.
- [Binomial theorem]: Use the binomial theorem to answer the following questions:
 - What is the coefficient of x^3 in the expansion of $(x+1)^{10}$?
 - What is the coefficient of x^3 in the expansion of $(x+2)^{10}$?
 - What is the coefficient of x^{98} in the expansion of $(x+1)^{100}$?
- [Binomial theorem]: Apply the binomial theorem to prove that for any given natural number n , $1 \leq n$, we have
$$1 = C(n,1) - C(n,2) + C(n,3) - C(n,4) + \dots + (-1)^{n+1} C(n,n)?$$
In other words, prove that $1 = -1 * \sum_{1 \leq i \leq n} (-1)^i C(n,i)$.
Hint: think about the expansion of $(-1+1)^n$.
- [Pigeonhole principle]: Prove that for any set X of at least 5 distinct natural numbers there must exist two numbers m and n in X such that $m > n$ and either $m+n$ or $m-n$ is divisible by 7.