Final exam: Part II **Due: 5:00pm Tuesday, May 20**

#12 (Congruence equations). Let $(a, b) = (289, 72)$. Do the following things:

(i). Apply the Euclidean algorithm to determine $gcd(289, 72)$, i.e. the greatest common divisor of 289 and 72.

(ii). Based on your work for (i) above, determine two integers x and y such that $289 * x + 72 * y = gcd(289, 72)$.

(iii). Is there an integer solution to the linear congruence equation $289 * x \equiv 1 \pmod{72}$? If so, give such an x. If not, explain why not.

(iv). Is there an integer solution to the linear congruence equation $289 * x \equiv 6 \pmod{72}$? If so, give such an x. If not, explain why not.

(v). Is there an integer solution to the linear congruence equation $72 * y \equiv 1 \pmod{289}$? If so, give such a y. If not, explain why not.

(vii). Is there an integer solution in the range of $[0, 289)$ (i.e. $0 \leq y < 289$) to the linear congruence equation $72 * y \equiv 6 \pmod{289}$? If so, give such a y. If not, explain why not.

#13 (Chinese remainder theorem). Find an integer x such that $0 \leq x < 105$ and x satisfies all of the following congruence equations: $x \equiv 1 \pmod 3$, $x \equiv 2 \pmod 5$, and $x \equiv 1 \pmod 7$.

#14. Consider any three finite sets X, Y and Z. Let's use S^c to denote the complement of a given set S, use the notation - to denote the set difference operation defined in Chapter 1 of the textbook (i.e. $X - Y = X \bigcap Y^c$), and use the notation $Power(S)$ to denote the set of all the subsets of a given set S. For each of the following statements, determine whether it is true or false. If you say it is false, give a counter example. If you say it is true, prove it based on the definitions of the set operations and the algebraic laws of set operations.

i $X \cup (Y - Z) = (X \cup Y) \cap (X \cup Z^c)$.
ii $Power(X \cup (Y - Z)) = Power(X \cup Y) \bigcap Power(X \cup Z^c)$.
iii $Power(X \cap (Y - Z)) = Power(X - Z) \bigcap Power(Y - Z)$.

#15. For each of the following statements, determine whether it is true or false. If you say it is false, give a counter example. If you say it is true, prove it. (i) Is it true that for any set X of 6 distinct non-negative integers there must exist two numbers m and n in X such that either m+n or m-n is divisible by 14? (ii) Is the statement above true for any set X of 7 distinct non-negative integers ? (iii) Is the statement above true for any set X of 8 distinct non-negative integers?

#X. (i) Rate your attendance this semester in a 0-10 scale. (ii) Rate your reading effort this semester in a 0-10 scale. (iii) What is the average amount of time you spent in a weekly reading assignment?