1. The principle of inclusion-exclusion tells us that given \( m \) finite sets, the size of their union can be determined by the information of the sizes of the individual sets and the sizes of the intersections of any \( 2, \ldots, m \) of these sets. For example, we have the following basic formula for the case of \( m=2 \) with two sets, \( A \) and \( B \): 
\[
|A \cup B| = |A| + |B| - |A \cap B|
\]
Using this basic formula for the case of \( m=2 \), we can derive the formula to the cases when \( m \) is 3 and 4 and so forth respectively based on our understanding of basic set operations and the basic formula for the case of \( m=2 \).

(i). To deal with the case of \( m=3 \) with 3 sets \( A, B, \) and \( C \), we can consider the union of \( A \cup B \cup C \) as the union of two sets: \( (A \cup B) \) and \( C \). Then we can apply the basic formula and get 
\[
|A \cup B \cup C| = |(A \cup B) \cup C| = |A \cup B| + |C| - |(A \cup B) \cap C|.
\]
Show how you can continue to apply the formula for the \( m=2 \) case together with the distributive law of set operations to derive the following formula for the \( m=3 \) case: 
\[
|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|.
\]

(ii). Similarly, to deal with the case of \( m=4 \) with 4 sets, \( A, B, C, \) and \( D \), we can consider \( A \cup B \cup C \cup D \) as the union of two sets, \( (A \cup B \cup C) \) and \( D \). Show how we can continue to apply the results we have for the \( m=2 \) case and that for the \( m=3 \) case derived above plus the distributive law of set operation to derive a similar formula for the \( m=4 \) case.

2. Consider the set of all students currently enrolled at Biola University as our universal set \( U \). We use \( F \), \( M \), and \( C \) to denote the following three subsets of \( U \): 
\[
F = \{ x | x \text{ is a female student at Biola.} \}, \quad M = \{ x | x \text{ takes math112 this semester.} \}, \quad \text{and } C = \{ x | x \text{ takes cs106 this semester.} \}.
\]
We are told that (i) the set of Biola students \( U \) is composed of 4000 students, 2050 female students plus 1950 male students, (ii) 29 students take math112 and 18 students take cs106, (iii) there are 10 female students in math112 and there are 4 female students in cs106, (iv) there are 10 students take both math112 and cs106, and (v) no female student takes both math112 and cs106.

(i). What are \( n(U) \), \( n(F) \), \( n(M) \), \( n(C) \), \( n(F \cap M) \), \( n(F \cap C) \), \( n(M \cap C) \), \( n(F \cap M \cap C) \) respectively?

(ii). Determine what is \( n(F \cup M \cup C) \) using the principle of inclusion-exclusion formula in part (i) of problem 1.

(iii). Determine the number of male Biola students that do not take math112 nor cs106 this semester. (Hint: These are simply the students that are not in \( F \), and not in \( M \), and not in \( C \). So the set of these students is simply the complement of \( F \cup M \cup C \). )

3. Apply the principle of inclusion and exclusion to determine the number of integers in the range of \([1, 10000]\) that are divisible by either 3, 5, 7, or 11.

4. Five guests a, b, c, d, and e attend a party and each of them bring a different for exchange. After the party, one by one they randomly pick a gift. (i) How many different ways are there to distribute the five different gifts to the five different guests? (ii) Among the different ways to distribute the gifts, how many of them will end up giving at least one of the gifts back to the guest who brought the gift in earlier? (iii) Assuming every way of distributing the gifts is equally likely to happen, what is the probability that after the party at least one of the guests would get his/her own gift back?