

Discrete Structures: Homework 8

Due: Thursday, May 5.

For each of the following statements, determine whether it is true or false. If you say it is false, give a counter example. If you say it is true, prove it based on facts and theorems in the book with well founded logical arguments.

- (1) Given a graph G where $v_5 \rightarrow v_2 \rightarrow v_7 \rightarrow v_6 \rightarrow v_4$ is a shortest path from vertex v_5 to vertex v_4 in a graph, then $v_2 \rightarrow v_7 \rightarrow v_6$ must be a shortest path from vertex v_2 to vertex v_6 .
- (2) If a graph is not a planar graph, then we need more than 4 colors to color the vertices so that no two adjacent vertices have the same color.
- (3) If a graph can be colored in two colors such that no two adjacent vertices have the same color, then it must be a planar graph.
- (4) If every vertex of a simple connected graph (of at least 3 vertices) has a degree of at least 6, then that graph can never be a planar graph.
- (5) A graph of n vertices and $n-1$ edges is always a tree.
- (6) A graph of n vertices and less than $n-1$ edges cannot be a connected graph.