Discrete Structures: Homework 8 Due: Thursday, May 15

1. Let p, q, r, and s be four propositions. Let $\bigvee$ denote the disjunction operator (i.e. logical OR), $\bigwedge$ denote the conjunction operator (i.e. logical AND), and $\neg$ denote the negation operator (i.e. logical NOT).

(i). Construct truth tables to demonstrate that the following compound propositions $(\neg p \bigwedge r) \bigvee (\neg p \bigwedge s) \bigvee (q \bigwedge r) \bigvee (q \bigwedge s)$ and $(\neg p \bigvee q) \bigwedge (r \bigvee s)$ are logically equivalent.

(ii). Prove (i) again by applying the laws of algebra of propositions on p.75.

(iii). Construct truth tables to demonstrate that the following compound propositions $(p \bigvee \neg r) \bigwedge (p \bigvee s) \bigwedge (q \bigvee \neg r) \bigwedge (q \bigvee s)$ and $(p \bigwedge q) \bigvee (\neg r \bigwedge s)$ are logically equivalent.

(iv). Prove (iii) again by applying the laws of algebra of propositions on p.75.

2. John defines that a year X is a *perfect year* if and only if ((X is divisible by 3) and (X is divisible by 400 or X is not divisible by 7)). Let P, D3, D7 and D400 denote the atomic propositions that a year X is a perfect year, is divisible by 3, by 7, by 400 respectively. Please translate John's definition of what is a perfect year into a compound logic proposition involving the atomic propositions above and the logical operations mentioned in chapter 4.

3. Mary defines that a year X is a *perfect year* if and only if it is not that (X is not divisible by 3 or (X is divisible by 7 but not by 400)). Let P, D3, D7 and D400 denote the atomic propositions that a year X is a perfect year, is divisible by 3, by 7, by 400 respectively. Please translate Mary's definition of what is a perfect year into a compound logic proposition involving the atomic propositions above and the logical operations mentioned in chapter 4.

4. Prove that John's compound logic proposition is logically equivalent to Mary's compound logic proposition by constructing truth tables.

5. Prove that John's compound logic proposition is logically equivalent to Mary's compound logic proposition by applying the laws of algebra of propositions on p.84.

6. The logic statements of perfect years in problem 2 and 3 above, actually can be much more precisely written if we use the universal quantifier $\forall$ and use P(X), D3(X), D7(X) and D400(X) to denote the propositional functions that a year X is a perfect year, is divisible by 3, by 7, by 400 respectively. Please rewrite the logic statements of John and Mary above using the universal quantifier and four propositional functions P(X), D3(X), D7(X) and D400(X).