Discrete Structures: Homework 5

#1. Consider the six statements below regarding any given finite sets X and Y and set operations (including the power set operation) on them. For each of the statements above, determine whether it is universally true for any given finite sets X and Y. If you think it is not, give a concrete counter example of X and Y in which the statement does not hold. If you think it is universally true for any given finite sets X and Y, provide a direct proof to show it is true.

Note that to prove that $A \subseteq B$ (set A is a subset of set B) is true using a direct proof, you need to show that if x belongs to A (i.e. $x \in A$) then x also belongs to B (i.e. $x \in B$). To prove $A = B$ using a direct proof, you need to prove both $A \subseteq B$ and $B \subseteq A$.

i. $P(X) \bigcap P(Y) \subseteq P(X \bigcap Y)$ (4 points)
ii. $P(X \bigcap Y) \subseteq P(X) \bigcap P(Y)$ (4 points)
iii. $P(X \bigcap Y) = P(X) \bigcap P(Y)$ (1 point)

iv. $P(X) \bigcup P(Y) \subseteq P(X \bigcup Y)$ (4 points)
v. $P(X \bigcup Y) \subseteq P(X) \bigcup P(Y)$ (4 points)
vi. $P(X \bigcup Y) = P(X) \bigcup P(Y)$ (1 point)

#2 [4 points]. For Problem #2, you can use the set identities (algebraic laws on sets) in Table 1 of Section 2.2 to help you derive the result. In Section 2.2 (p. 128), the formula $|B_1 \bigcup B_2| = |B_1| + |B_2| - |B_1 \bigcap B_2|$ tells us that given any two finite sets B_1 and B_2 the number of elements in $B_1 \bigcup B_2$ is always equal to the number of elements in B_1 plus the number of elements in B_2 minus the number of elements in $B_1 \bigcap B_2$. Use this formula as a premise and prove (using direct proof) that for **any three** finite sets B_1, B_2, and B_3, we always have

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\begin{aligned}
&|B_1 \bigcup B_2 \bigcup B_3| \\
&= |B_1| + |B_2| + |B_3| \\
&-|B_1 \bigcap B_2| - |B_1 \bigcap B_3| - |B_2 \bigcap B_3| \\
&+|B_1 \bigcap B_2 \bigcap B_3|.
\end{aligned}
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Hint: View $B_2 \bigcup B_3$ as B_2' and reason about $|B_1 \bigcup B_2'|$.