Discrete Structures: Homework 4

For each of the twelve statements in the end below, determine whether it is universally true for any given finite sets X, Y and Z. If you think it is not universally true for any given finite sets X, Y and Z, give a concrete counter example (i.e. showing three specific sets X, Y and Z) in which the statement does not hold. If you think it is universally true for any given finite sets X, Y and Z, write a direct proof to prove it.

Note that to prove that $A \subseteq B$ (set A is a subset of set B) is true using a direct proof, you need to show that if x belongs to A (i.e. $x \in A$) then x also belongs to B (i.e. $x \in B$). To prove $A = B$ using a direct proof, you need to prove both $A \subseteq B$ and $B \subseteq A$.

Also note that for this homework we want you to prove things based on logic (such as the algebraic laws of propositional logic in Chapter 1) and definitions of sets and set operations. To gain points for this homework, you can not directly use the set identities in Table 1 of Section 2.2 to prove the equality of sets.

1. $(X - Z) \bigcap (Y - Z) \subseteq (X \bigcap Y) - Z$ (3 points)
2. $(X \bigcap Y) - Z \subseteq (X - Z) \bigcap (Y - Z)$ (3 points)
3. $(X \bigcap Y) - Z = (X - Z) \bigcap (Y - Z)$ (1 point)

4. $(X - Z) \bigcup (Y - Z) \subseteq (X \bigcup Y) - Z$ (3 points)
5. $(X \bigcup Y) - Z \subseteq (X - Z) \bigcup (Y - Z)$ (3 points)
6. $(X \bigcup Y) - Z = (X - Z) \bigcup (Y - Z)$ (1 point)

7. $(X \bigcap Y) \bigcap (X - Z) \subseteq X \bigcap (Y - Z)$ (3 points)
8. $X \bigcap (Y - Z) \subseteq (X \bigcap Y) \bigcap (X - Z)$ (3 points)
9. $X \bigcap (Y - Z) = (X \bigcap Y) \bigcap (X - Z)$ (1 point)

10. $(X \bigcup Y) \bigcup (X - Z) \subseteq X \bigcup (Y - Z)$ (3 points)
11. $X \bigcup (Y - Z) \subseteq (X \bigcup Y) \bigcup (X - Z)$ (3 points)
12. $X \bigcup (Y - Z) = (X \bigcup Y) \bigcup (X - Z)$ (1 point)