

The vehicle starts at the location of the fuel station S_1 and travels along a fixed path that passes through the fuel stations S_1 , S_2 , ..., S_{N-1} , S_N in order. The fuel station S_N is the very last fuel station before reaching the final destination.

Coefficients (i.e. parameters)

- *capacity*: Tank capacity of the vehicle (in gallons).
- *initial*: The initial amount of fuel in the tank of the vehicle (in gallons).
- N: the number of the stations (stations $S_1, S_2, ..., S_N$) on a given fixed path.
- For each j in 1 to N, we have the following additional information associated with station S_j
 - c_j : the fuel cost per gallon at the *j* th station S_j .
 - g_j : the amount of fuel consumed (in gallons) to go from station S_j to the next station (or to the final destination from the very last station S_{N_j} .

Variables about refueling decisions at stations S_j's

- Variable Y_j : the amount of fuel to fill in at station S_j .
- If Y_j is 0, it means the vehicle simply passes by without refueling at station S_j .
- Note that (i) you can never refuel the tank to go beyond the tank capacity and (ii) when you leave station S_j , you should have enough fuel to reach the next station (or to reach the destination when leaving S_N).
- A feasible refueling policy $\langle Y_1, Y_2, ..., Y_N \rangle$ ensures that the amount of fuel in the tank should never go below 0 and should never go beyond the tank capacity throughout the entire trip.
- An optimal refueling policy < Y_1 , Y_2 , ..., Y_N > is a feasible refueling policy that minimizes the total fuel cost.

Operational objective:

Determine Y_j (the amount of fuel to fill in at station S_j) for each j in 1 to N to minimize the total the fuel cost subject to the constraints that the amount of fuel in the tank should never go below 0 and should never go beyond the tank capacity. In other words, **determine an optimal refueling policy** $< Y_1, Y_2, ..., Y_N >$ for the trip.

Constraints

- You should never refuel the tank to a level beyond the tank capacity. Question: How do you model the fuel level when leaving each station S_j in terms of the variables Y_j 's and all the related coefficients?
- You should never have an empty tank in the middle of the journey between two stations. In other words, when you leave every station S_j , the fuel level must be greater than or equal to g_j .
- You cannot fill in a negative amount of fuel, i.e. not allowed to sell the fuel to the stations.

An alternative way to build equivalent linear programs and a generic AMPL model:

You can surely develop linear programs and even a generic AMPL model for solving the simple vehicle refueling problem purely based on the variables Y_j 's. Alternatively some people find it helpful to introduce the additional variables X_j 's and Z_j 's in the following to simplify the descriptions of constraints when developing linear programs or a generic AMPL model for solving the problem.

Coefficients (i.e. parameters)

- *capacity*: Tank capacity of the vehicle (in gallons).
- *initial*: The initial amount of fuel in the tank of the vehicle (in gallons).
- N: the number of the stations (stations $S_1, S_2, ..., S_N$) on a given fixed path.
- For each j in 1 to N, we have the following additional information associated with station S_j
 - c_i : the fuel cost per gallon at the *j* th station S_i
 - g_j : the amount of fuel consumed (in gallons) to go from station S_j to the next station (or to the final destination from the very last station S_{N_j} .

Variable and constraints:

- Variable X_j : the amount of fuel in the tank when the vehicle just arrives at station S_j without doing any refueling operation there yet.
- Variable Y_j : the amount of fuel to fill in at station S_j.
- Variable Z_j : the amount of fuel in the tank when the vehicle is going to leave station S_j (possibly after a refueling operation there).
- **Constraints:** all X_i , Y_j , Z_j must be non-negative.
- **Constraint:** $X_1 = initial.$
- Constraints: $X_j = Z_{j-1} g_{j-1}$ for every Station S_i where i > 1.
- **Constraints:** $Z_i = X_i + Y_i$ for every Station S_i .
- **Constraints:** $Z_j \ge g_j$ for every Station S_i .
- **Constraints:** $Z_j \leq capacity$ for every Station S_i .