##
The Fixed Path Vehicle Refueling Problem
MATH 333, Operations Research.
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The vehicle starts at the location of the fuel station S_1 and travels along a fixed path that passes through the fuel stations S_1, S_2, …, S_{N-1}, S_N in order. The fuel station S_N is the very last fuel station before reaching the final destination.

Coefficients (i.e. parameters)

- Tank capacity of the vehicle (in gallons).
- The initial amount of fuel in the tank of the vehicle (in gallons).
- N: the number of the stations (stations S_1, S_2, …, S_N) on a given fixed path.
- For each j in 1 to N, we have the following additional information associated with station S_j
  - c_j: the fuel cost per gallon at the *j* th station S_j.
  - g_j: the amount of fuel consumed (in gallons) to go from station S_j to the next station (or to the final destination from the very last station S_N).

Variables about refueling decisions at stations S_j's

- Y_j : the amount of fuel to fill in at station S_j.
- If Y_j is 0, it means the vehicle simply passes by without refueling at station S_j .
- **Note that** (i) you can never refuel the tank to go beyond the tank capacity and (ii) when you leave station S_j, you should have enough fuel to reach the next station (or to reach the destination when leaving S_N).
- **A feasible refueling policy** < Y_1 **,** Y_2 **, …,**Y_N> **ensures** that the amount of fuel in the tank should never go below 0 and should never go beyond the tank capacity throughout the entire trip.
- **An optimal refueling policy** < Y_1 **,** Y_2 **, …,**Y_N> **is a feasible refueling policy that minimizes the total fuel cost.**

Other variables you may also consider in the context of station S_j

- X_j : the amount of fuel in the tank when the vehicle just arrives at station S_j without doing any refueling operation there.
- Z_j : the amount of fuel in the tank when the vehicle is going to leave station S_j (possibly after a refueling operation there).
- **Note that** (i) X_1 is determined by the initial amount of fuel in the tank of the vehicle, (ii) Z_j is determined by X_j and Y_j , and (iii) X_{j+1} is determined by Z_j and g_j .

Operational objective:

Determine Y_j (the amount of fuel to fill in at station S_j) for each j in 1 to N to minimize the total the fuel cost subject to the constraints that the amount of fuel in the tank should never go below 0 and should never go beyond the tank capacity. In other words, **determine an optimal refueling policy** $< Y_1 , Y_2 , \ldots, Y_N >$ **for the trip.**

Constraints

- You should never refuel the tank to a level beyond the tank capacity.
- You should never have an empty tank in the middle of the journey between two stations. In other words, when you leave a station S_j, you should have enough fuel to reach the next station (or to reach the destination when leaving S_N).