Homework #6 Mixed-strategy Nash equilibria in zero-sum games: an ad hoc case

	R	P	S
R	0, 0	-2, 2	3, -3
P	2, -2	0, 0	-1, 1
S	-3, 3	1, -1	0, 0

Consider the game above as a variant of rock paper scissors:

- As usual, ***Rock*** beats ***scissors, paper*** beats ***rock,*** and ***scissors*** beats ***paper***.
- When a player plays ***rock*** and wins, he/she is the winner and gets a reward of $3 while the other player needs to pay $3 to the winner.
- When a player plays ***paper*** and wins, he/she is the winner and gets a reward of $2 while the other player needs to pay $2 to the winner.
- When a player plays ***scissors*** and wins, he/she is the winner and gets a reward of $1 while the other player needs to pay $1 to the winner.
- When there is a tie, the payoff is 0 for both players.

Problems:

1. When restricted to pure strategies only, do we have a Nash equilibrium? If so, determine the Nash equilibrium in pure strategies. If not, explains why there is none.
2. Is there a mixed strategy y for **player 1** such that for **player 2** R and P and S are all best responses to y? If so, please determine and show such a mixed strategy y for player 1. Similarly, is there a mixed strategy x for **player 2** such that **for player 1** R and P and S are all best responses to x? If so, please determine and show such a mixed strategy x for player 1.
3. Show that < y, x> is a Nash equilibrium when there do exist a mixed strategy y for **player 1** and a mixed strategy x for **player 2** with the properties described above in 2. In other words, show (i) y is a best response from **player 1** when player 2 plays x and (ii) x is a best response from **player 2** when player 1 plays y. **Note that in general this kind of mixed strategies do not always exist for a zero-sum game with two players, especially when each player has a different number of pure strategies to consider.**