Consider the game above as a variant of rock paper scissors:

- As usual, Rock beats scissors, paper beats rock, and scissors beats paper.
- When a player plays rock and wins, he/she is the winner and gets a reward of $3 while the other player needs to pay $3 to the winner.
- When a player plays paper and wins, he/she is the winner and gets a reward of $2 while the other player needs to pay $2 to the winner.
- When a player plays scissors and wins, he/she is the winner and gets a reward of $1 while the other player needs to pay $1 to the winner.
- When there is a tie, the payoff is 0 for both players.

Problems:

1. When restricted to pure strategies only, do we have a Nash equilibrium? If so, determine the Nash equilibrium in pure strategies. If not, explains why there is none.
2. Is there a mixed strategy y for player 1 such that for player 2 R and P and S are all best responses to y? If so, please determine and show such a mixed strategy y for player 1. Similarly, is there a mixed strategy x for player 2 such that for player 1 R and P and S are all best responses to x? If so, please determine and show such a mixed strategy x for player 1.
3. Show that < y, x > is a Nash equilibrium when there do exist a mixed strategy y for player 1 and a mixed strategy x for player 2 with the properties described above in 2. In other words, show (i) y is a best response from player 1 when player 2 plays x and (ii) x is a best response from player 2 when player 1 plays y. Note that in general this kind of mixed strategies do not always exist for a zero-sum game with two players, especially when each player has a different number of pure strategies to consider.