Test #4

Integrity rules for the open-book take-home test:

- **Time and submission:** Allocate up to **4 hours (240 minutes)** to do the test after the class today 05/19/2015, including time spent in looking up things from the book/notes/homework. Put down your answers in a word document and submit it under Canvas by **11:50am 05/20/2015**.

- **Resources allowed:** During the test (from the point of announcement of the problem set to the point you submit your work), **the only resources allowed are the textbook and your class notes, and your earlier homework.** AMPL is allowed for computation if needed.

- **No collaboration or additional resources:** During the test, you **should not** discuss the test with others online or offline and you **should not search for** additional resources or clues from the internet for doing the test.

- **Violation of the integrity rules is viewed as cheating in the test.**

#1. For problem #1, let’s assume the unit production cost is the same for all companies involved.

Consider the three models of markets in chapter 6 of the game theory book regarding the production of homogeneous products: (i) the **Cournot** model of duopoly with two companies setting their own production amounts $Q_1$ and $Q_2$ at the same time, (ii) the **cartel** model with the companies forming a single cartel to monopolize the market to produce the joint amount of $Q=Q_1+Q_2$, and (iii) the **Stackelburg** model of duopoly with company 1 setting its production amount $Q_1$ first and then company 2 setting the its production amount $Q_2$ after knowing company 1’s amount $Q_1$.

Let $Q_1$, $Q_2$ be the amounts produced by company 1 and company 2. Let $Q=Q_1+Q_2$ refer the **total amount** of supply in the market combined from both companies. Let $P$ denote the market price. Let $Net_1$, $Net_2$ be the net profits (income – production cost) for company 1 and company 2 respectively. Let $Net= Net_1+ Net_2$ refer the **joint net profit** combined from both companies.

Let $P= 20 – 2Q$ be the relationship between $Q$ and $P$, and let unit production cost $c$ be 1. For each of the following statements, determine whether it is true or false. **If it is false, please explain why it is false.**

I. **The market will see more total supply $Q$ under the Cournot model than under the Cartel model.** Actually the former is $4/3$ times that of the later.
II. The market will see a higher price $P$ under the Cournot model than that under the Cartel model.

III. The join net profit $Net = Net1 + Net2$ is higher under the Cournot model than under the Cartel model.

IV. The market will see more total supply $Q$ under the Cournot model than under the Stackelburg model.

V. The market will see a higher price $P$ under the Cournot model than under the Stackelburg model.

VI. The amount $Q2$ for company #2 under the Cournot model is lower than under the Stackelburg model while the amount $Q1$ for company #1 under the Cournot model is higher than under the Stackelburg model.

VII. The net profit $Net2$ for company #2 under the Cournot model is lower than under the Stackelburg model while $Net1$ for company #1 under the Cournot model is higher than under the Stackelburg model.

VIII. The market will see more total supply $Q$ under the Stackelburg model than under the Cartel model. Actually the former is $3/2$ times that of the later.

IX. The market will see a higher price $P$ under the Stackelburg model than that under the Cartel model.

X. The join net profit $Net = Net1 + Net2$ is higher under the Stackelburg model than under the Cartel model.

#2. Consider the Cournot model of duopoly in Problem #1. Let $P = 20 - 2Q$ be the relationship between $Q$ and $P$ again. However, instead of the same production cost per unit, now let the production cost be $0.5$ dollar per unit for company 1 while the production cost be $1.5$ dollar per unit for company 2. What would be $Q1$, $Q2$ now when they reach the Nash equilibrium? What would be the corresponding market price $P$?

#3. One version of the commons problem investigated in Chapter 7 of the textbook has the setting of two persons involved in sharing common resources through 2 periods and the book describes the Nash equilibrium solution reached under the setting. In the class, we have shown how you can generalize the analysis to determine the Nash equilibrium solution regarding how much of the initial resources will be consumed by each person in each of the 3 periods when the setting is generalized to two persons involved in sharing common resources through 3 periods (instead of 2 periods).

Your task now: Show how you can generalize the analysis to determine the Nash equilibrium solution regarding how much of the initial resources will be consumed by each person in each of the 3 periods when the setting is generalized to $n$ persons involved in sharing common resources through 3 periods. Note: Instead of just putting down the results, you should describe how you get the answers.
#4. One version of the **commons** problem investigated in Chapter 7 of the textbook has the setting of **n persons** involved in sharing common resources through **2 periods** and the book describes the **social optimality solution** reached under the setting.

**Your task now:** Show how you can generalize the analysis to determine the **social optimality solution** regarding how much of the initial resources will be consumed by each person in each of the **3 periods** when the setting is generalized to **n persons** involved in sharing common resources through **3 periods**. **Note:** Instead of just putting down the results, you should describe how you get the answers.

**Submission:**

- **Zip file:** You can put down your answers entirely in a word document, or work it out entirely on paper and then scan it into an image file (jpg or other formats), or a combination of both. **But in the end, please compress them into a single zip file and upload it under Canvas.**